6.4 Rhombuses, Rectangles, and Squares

What *you should learn*

GOAL 1 Use properties of sides and angles of rhombuses, rectangles, and squares.

GOAL 2 Use properties of diagonals of rhombuses, rectangles, and squares.

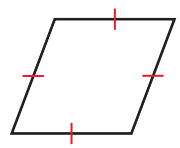
Why *you should learn it*

▼ To simplify **real-life** tasks, such as checking whether a theater flat is rectangular in **Example 6**.

REAL LIFE

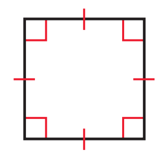
GOAL 1 PROPERTIES OF SPECIAL PARALLELOGRAMS

In this lesson you will study three special types of parallelograms: *rhombuses*, *rectangles*, and *squares*.



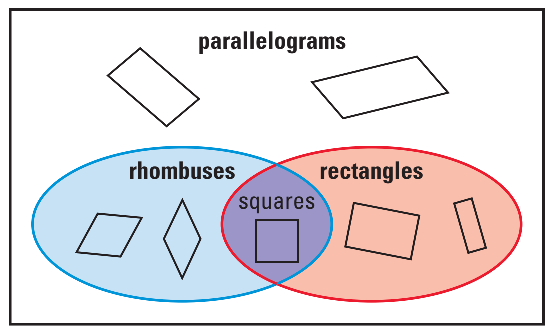
A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

The *Venn diagram* at the right shows the relationships among parallelograms, rhombuses, rectangles, and squares. Each shape has the properties of every group that it belongs to. For instance, a square is a rectangle, a rhombus, and a parallelogram, so it has all of the properties of each of those shapes.

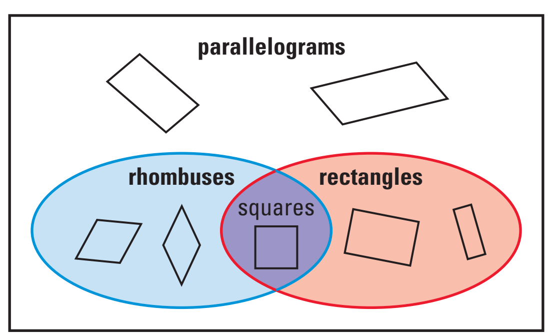


EXAMPLE 1 *Describing a Special Parallelogram*

Decide whether the statement is *always*, *sometimes*, or *never* true.

a. A rhombus is a rectangle.

b. A parallelogram is a rectangle.



SOLUTION

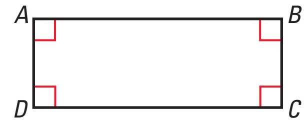
a. The statement is *sometimes* true. In the Venn diagram, the regions for rhombuses and rectangles overlap. If the rhombus is a square, it is a rectangle.

b. The statement is *sometimes* true. Some parallelograms are rectangles. In the Venn diagram, you can see that some of the shapes in the parallelogram box are in the region for rectangles, but many aren't.

EXAMPLE 2 Using Properties of Special Parallelograms

Logical Reasoning

ABCD is a rectangle. What else do you know about *ABCD*?



SOLUTION

Because *ABCD* is a rectangle, it has four right angles by the definition. The definition also states that rectangles are parallelograms, so *ABCD* has all the properties of a parallelogram:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent and consecutive angles are supplementary.
- Diagonals bisect each other.

A rectangle is defined as a *parallelogram* with four right angles. But *any quadrilateral* with four right angles is a rectangle because any quadrilateral with four right angles is a parallelogram. In Exercises 48–50 you will justify the following corollaries to the definitions of rhombus, rectangle, and square.

STUDENT HELP

Look Back
For help with biconditional statements, see p. 80.

COROLLARIES ABOUT SPECIAL QUADRILATERALS

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

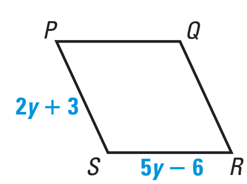
SQUARE COROLLARY

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

You can use these corollaries to prove that a quadrilateral is a rhombus, rectangle, or square without proving first that the quadrilateral is a parallelogram.

EXAMPLE 3 Using Properties of a Rhombus

In the diagram at the right, *PQRS* is a rhombus. What is the value of *y*?



SOLUTION

All four sides of a rhombus are congruent, so $RS = PS$.

$5y - 6 = 2y + 3$	Equate lengths of congruent sides.
$5y = 2y + 9$	Add 6 to each side.
$3y = 9$	Subtract $2y$ from each side.
$y = 3$	Divide each side by 3.

GOAL 2 USING DIAGONALS OF SPECIAL PARALLELOGRAMS

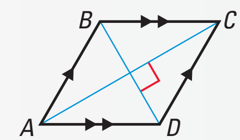
The following theorems are about diagonals of rhombuses and rectangles. You are asked to prove Theorems 6.12 and 6.13 in Exercises 51, 52, 59, and 60.

THEOREMS

THEOREM 6.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

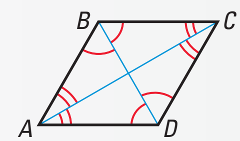
$ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



THEOREM 6.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

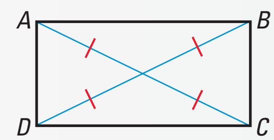
$ABCD$ is a rhombus if and only if
$\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$ and
$\overline{BD}$ bisects $\angle ADC$ and $\angle CBA$.



THEOREM 6.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

$ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.



You can rewrite Theorem 6.11 as a conditional statement and its converse.

Conditional statement: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Converse: If a parallelogram is a rhombus, then its diagonals are perpendicular.

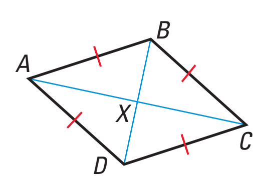
To prove the theorem, you must prove both statements.

EXAMPLE 4 *Proving Theorem 6.11*

Write a paragraph proof of the converse above.

GIVEN ▶ $ABCD$ is a rhombus.

PROVE ▶ $\overline{AC} \perp \overline{BD}$



SOLUTION

Paragraph Proof $ABCD$ is a rhombus, so $\overline{AB} \cong \overline{CB}$. Because $ABCD$ is a parallelogram, its diagonals bisect each other so $\overline{AX} \cong \overline{CX}$ and $\overline{BX} \cong \overline{BX}$. Use the SSS Congruence Postulate to prove $\triangle AXB \cong \triangle CXB$, so $\angle AXB \cong \angle CXB$. Then, because $\overline{AC}$ and $\overline{BD}$ intersect to form congruent adjacent angles, $\overline{AC} \perp \overline{BD}$.

STUDENT HELP

INTERNET
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 5 Coordinate Proof of Theorem 6.11

Proof

In Example 4, a paragraph proof was given for part of Theorem 6.11. Write a coordinate proof of the original conditional statement.

GIVEN ▶ $ABCD$ is a parallelogram, $\overline{AC} \perp \overline{BD}$.

PROVE ▶ $ABCD$ is a rhombus.

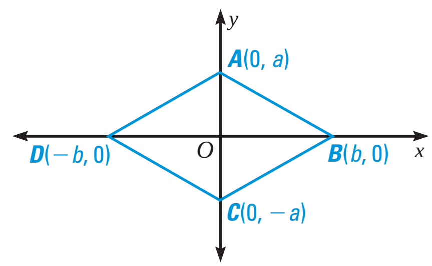
SOLUTION

Assign coordinates Because $\overline{AC} \perp \overline{BD}$, place $ABCD$ in the coordinate plane so $\overline{AC}$ and $\overline{BD}$ lie on the axes and their intersection is at the origin.

Let $(0, a)$ be the coordinates of A, and let $(b, 0)$ be the coordinates of B.

Because $ABCD$ is a parallelogram, the diagonals bisect each other and $OA = OC$. So, the coordinates of C are $(0, -a)$.

Similarly, the coordinates of D are $(-b, 0)$.



Find the lengths of the sides of $ABCD$. Use the Distance Formula.

$$AB = \sqrt{(b - 0)^2 + (0 - a)^2} = \sqrt{b^2 + a^2}$$

$$BC = \sqrt{(0 - b)^2 + (-a - 0)^2} = \sqrt{b^2 + a^2}$$

$$CD = \sqrt{(-b - 0)^2 + [0 - (-a)]^2} = \sqrt{b^2 + a^2}$$

$$DA = \sqrt{[0 - (-b)]^2 + (a - 0)^2} = \sqrt{b^2 + a^2}$$

▶ All of the side lengths are equal, so $ABCD$ is a rhombus.

FOCUS ON APPLICATIONS

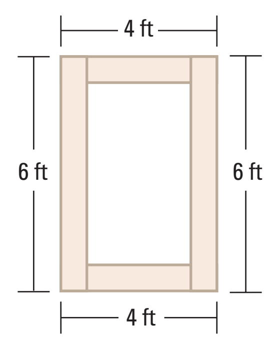
CARPENTRY If a screen door is not rectangular, you can use a piece of hardware called a *turnbuckle* to shorten the longer diagonal until the door is rectangular.

EXAMPLE 6 Checking a Rectangle

CARPENTRY You are building a rectangular frame for a theater set.

a. First, you nail four pieces of wood together, as shown at the right. What is the shape of the frame?

b. To make sure the frame is a rectangle, you measure the diagonals. One is 7 feet 4 inches and the other is 7 feet 2 inches. Is the frame a rectangle? Explain.



SOLUTION

a. Opposite sides are congruent, so the frame is a parallelogram.

b. The parallelogram is not a rectangle. If it were a rectangle, the diagonals would be congruent.

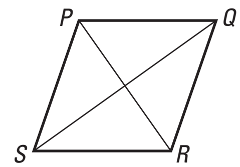
GUIDED PRACTICE

Vocabulary Check ✓

1. What is another name for an *equilateral quadrilateral*?

Concept Check ✓

2. Theorem 6.12 is a biconditional statement. Rewrite the theorem as a conditional statement and its converse, and tell what each statement means for parallelogram *PQRS*.



Skill Check ✓

Decide whether the statement is *sometimes*, *always*, or *never* true.

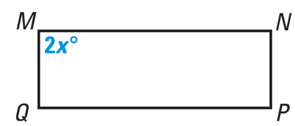
3. A rectangle is a parallelogram.
4. A parallelogram is a rhombus.
5. A rectangle is a rhombus.
6. A square is a rectangle.

Which of the following quadrilaterals have the given property?

7. All sides are congruent.
8. All angles are congruent.
9. The diagonals are congruent.
10. Opposite angles are congruent.

A. Parallelogram
B. Rectangle
C. Rhombus
D. Square

11. *MNPQ* is a rectangle. What is the value of x?



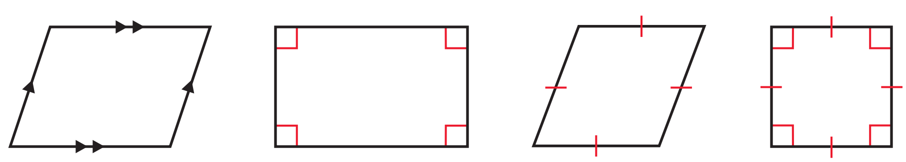
PRACTICE AND APPLICATIONS

STUDENT HELP
Extra Practice to help you master skills is on p. 814.

RECTANGLE **For any rectangle *ABCD*, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a sketch and explain your answer.**

12. $\angle A \cong \angle B$
13. $\overline{AB} \cong \overline{BC}$
14. $\overline{AC} \cong \overline{BD}$
15. $\overline{AC} \perp \overline{BD}$

PROPERTIES **List each quadrilateral for which the statement is true.**



16. It is equiangular.
17. It is equiangular and equilateral.
18. The diagonals are perpendicular.
19. Opposite sides are congruent.
20. The diagonals bisect each other.
21. The diagonals bisect opposite angles.

PROPERTIES **Sketch the quadrilateral and list everything you know about it.**

22. parallelogram *FGHI*
23. rhombus *PQRS*
24. square *ABCD*

STUDENT HELP
HOMEWORK HELP
Example 1: Exs. 12–15, 27–32
Example 2: Exs. 27–32, 51
Example 3: Exs. 33–43
Example 4: Exs. 44–52
Example 5: Exs. 55–60
Example 6: Exs. 61, 62

LOGICAL REASONING **Give another name for the quadrilateral.**

25. equiangular quadrilateral

26. regular quadrilateral

RHOMBUS **For any rhombus *ABCD*, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a sketch and explain your answer.**

27. $\angle A \cong \angle C$

28. $\angle A \cong \angle B$

29. $\angle ABD \cong \angle CBD$

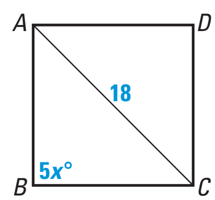
30. $\overline{AB} \cong \overline{BC}$

31. $\overline{AC} \cong \overline{BD}$

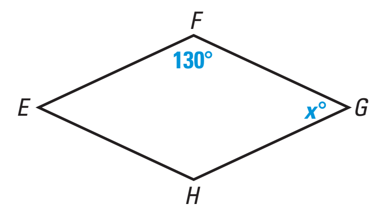
32. $\overline{AD} \cong \overline{CD}$

USING ALGEBRA **Find the value of *x*.**

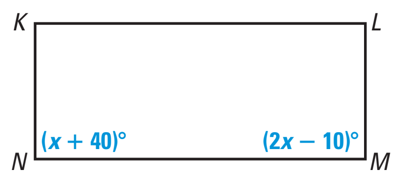
33. *ABCD* is a square.



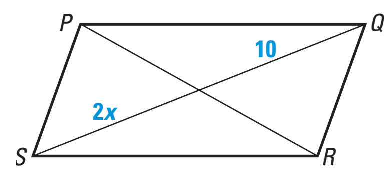
34. *EFGH* is a rhombus.



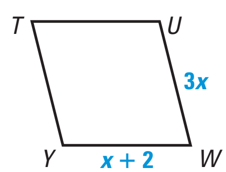
35. *KLMN* is a rectangle.



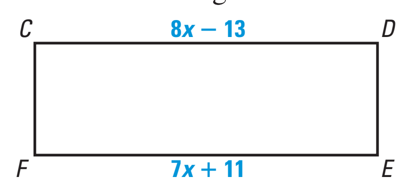
36. *PQRS* is a parallelogram.



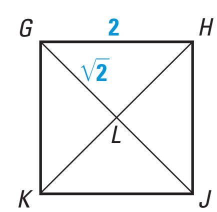
37. *TUWY* is a rhombus.



38. *CDEF* is a rectangle.



COMPLETING STATEMENTS ***GHJK* is a square with diagonals intersecting at *L*. Given that *GH* = 2 and *GL* = $\sqrt{2}$, complete the statement.**



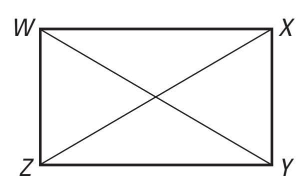
39. $HK = \underline{\ \ ?\ \ }$

40. $m\angle KLJ = \underline{\ \ ?\ \ }$

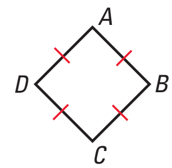
41. $m\angle HJG = \underline{\ \ ?\ \ }$

42. Perimeter of $\triangle HJK = \underline{\ \ ?\ \ }$

43. **USING ALGEBRA** *WXYZ* is a rectangle. The perimeter of $\triangle XYZ$ is 24. $XY + YZ = 5x - 1$ and $XZ = 13 - x$. Find *WY*.



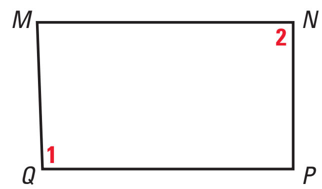
44. **LOGICAL REASONING** What additional information do you need to prove that $ABCD$ is a square?



PROOF **In Exercises 45 and 46, write any kind of proof.**

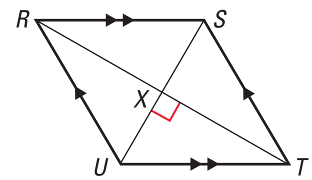
45. **GIVEN ▶** $\overline{MN} \parallel \overline{PQ}$, $\angle 1 \not\cong \angle 2$

PROVE ▶ $\overline{MQ}$ is not parallel to $\overline{PN}$.

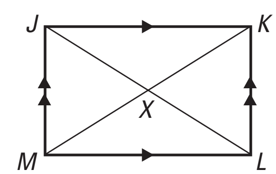


46. **GIVEN ▶** $RSTU$ is a $\square$, $\overline{SU} \perp \overline{RT}$

PROVE ▶ $\angle STR \cong \angle UTR$



47. **BICONDITIONAL STATEMENTS** Rewrite Theorem 6.13 as a conditional statement and its converse. Tell what each statement means for parallelogram $JKLM$.



LOGICAL REASONING Write the corollary as a conditional statement and its converse. Then explain why each statement is true.

48. Rhombus corollary

49. Rectangle corollary

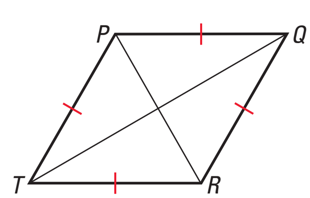
50. Square corollary

PROVING THEOREM 6.12 **Prove both conditional statements of Theorem 6.12.**

51. **GIVEN ▶** $PQRT$ is a rhombus.

PROVE ▶ $\overline{PR}$ bisects $\angle TPQ$ and $\angle QRT$. $\overline{TQ}$ bisects $\angle PTR$ and $\angle RQP$.

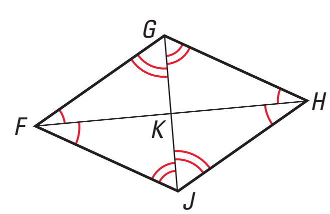
Plan for Proof To prove that $\overline{PR}$ bisects $\angle TPQ$ and $\angle QRT$, first prove that $\triangle PRQ \cong \triangle PRT$.



52. **GIVEN ▶** $FGHJ$ is a parallelogram. $\overline{FH}$ bisects $\angle JFG$ and $\angle GHJ$. $\overline{JG}$ bisects $\angle FJH$ and $\angle HGF$.

PROVE ▶ $FGHJ$ is a rhombus.

Plan for Proof Prove $\triangle FHJ \cong \triangle FHG$ so $\overline{JH} \cong \overline{GH}$. Then use the fact that $\overline{JH} \cong \overline{FG}$ and $\overline{GH} \cong \overline{FJ}$.



CONSTRUCTION **Explain how to construct the figure using a straightedge and a compass. Use a definition or theorem from this lesson to explain why your method works.**

53. a rhombus that is not a square

54. a rectangle that is not a square

COORDINATE GEOMETRY It is given that *PQRS* is a parallelogram. Graph ▱*PQRS*. Decide whether it is a *rectangle*, a *rhombus*, a *square*, or *none of the above*. Justify your answer using theorems about quadrilaterals.

55. $P(3, 1)$, $Q(3, -3)$, $R(-2, -3)$, $S(-2, 1)$

56. $P(5, 2)$, $Q(1, 9)$, $R(-3, 2)$, $S(1, -5)$

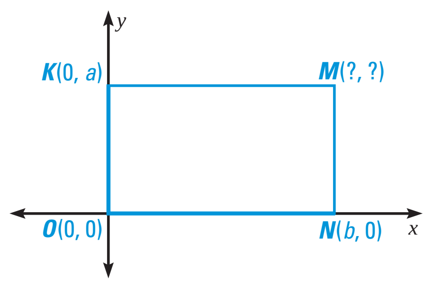
57. $P(-1, 4)$, $Q(-3, 2)$, $R(2, -3)$, $S(4, -1)$

58. $P(5, 2)$, $Q(2, 5)$, $R(-1, 2)$, $S(2, -1)$

COORDINATE PROOF OF THEOREM 6.13 In Exercises 59 and 60, you will complete a coordinate proof of one conditional statement of Theorem 6.13.

GIVEN ▶ *KMNO* is a rectangle.

PROVE ▶ $\overline{OM} \cong \overline{KN}$

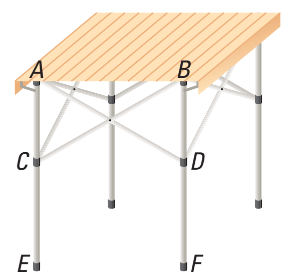


Because $\angle O$ is a right angle, place *KMNO* in the coordinate plane so *O* is at the origin, $\overline{ON}$ lies on the *x*-axis and $\overline{OK}$ lies on the *y*-axis. Let the coordinates of *K* be $(0, a)$ and let the coordinates of *N* be $(b, 0)$.

59. What are the coordinates of *M*? Explain your reasoning.

60. Use the Distance Formula to prove that $\overline{OM} \cong \overline{KN}$.

PORTABLE TABLE The legs of the table shown at the right are all the same length. The cross braces are all the same length and bisect each other.



61. Show that the edge of the tabletop $\overline{AB}$ is perpendicular to legs $\overline{AE}$ and $\overline{BF}$.

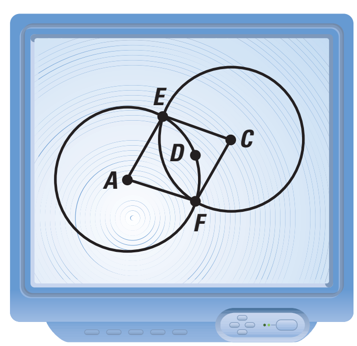
62. Show that $\overline{AB}$ is parallel to $\overline{EF}$.

STUDENT HELP

INTERNET

SOFTWARE HELP
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

TECHNOLOGY In Exercises 63–65, use geometry software.



Draw a segment $\overline{AB}$ and a point *C* on the segment. Construct the midpoint *D* of $\overline{AB}$. Then hide $\overline{AB}$ and point *B* so only points *A*, *D*, and *C* are visible.

Construct two circles with centers *A* and *C* using the length $\overline{AD}$ as the radius of each circle. Label the points of intersection *E* and *F*. Draw $\overline{AE}$, $\overline{CE}$, $\overline{CF}$, and $\overline{AF}$.

63. What kind of shape is *AECF*? How do you know? What happens to the shape as you drag *A*? drag *C*?

64. Hide the circles and point *D*, and draw diagonals $\overline{EF}$ and $\overline{AC}$. Measure $\angle EAC$, $\angle FAC$, $\angle AEF$, and $\angle CEF$. What happens to the measures as you drag *A*? drag *C*?

65. Which theorem does this construction illustrate?

Test Preparation

66. **MULTIPLE CHOICE** In rectangle *ABCD*, if $AB = 7x - 3$ and $CD = 4x + 9$, then $x = \underline{?}$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

67. **MULTIPLE CHOICE** In parallelogram *KLMN*, $KM = LN$, $m\angle KLM = 2xy$, and $m\angle LMN = 9x + 9$. Find the value of y.

(A) 9 (B) 5 (C) 18

(D) 10 (E) Cannot be determined.

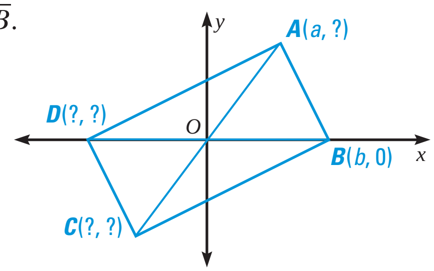
68. *Writing* Explain why a parallelogram with one right angle is a rectangle.

★ Challenge

COORDINATE PROOF OF THEOREM 6.13 **Complete the coordinate proof of one conditional statement of Theorem 6.13.**

GIVEN ▶ *ABCD* is a parallelogram, $\overline{AC} \cong \overline{DB}$.

PROVE ▶ *ABCD* is a rectangle.



Place *ABCD* in the coordinate plane so $\overline{DB}$ lies on the x-axis and the diagonals intersect at the origin. Let the coordinates of B be $(b, 0)$ and let the x-coordinate of A be a as shown.

69. Explain why $OA = OB = OC = OD$.

70. Write the y-coordinate of A in terms of a and b. Explain your reasoning.

71. Write the coordinates of C and D in terms of a and b. Explain your reasoning.

72. Find and compare the slopes of the sides to prove that *ABCD* is a rectangle.

EXTRA CHALLENGE
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MIXED REVIEW

USING THE SAS CONGRUENCE POSTULATE **Decide whether enough information is given to determine that $\triangle ABC \cong \triangle DEF$.** (Review 4.3)

73. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$

74. $\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{CA}$, $\angle A \cong \angle D$

75. $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$

76. $\overline{EF} \cong \overline{BC}$, $\overline{DF} \cong \overline{AB}$, $\angle A \cong \angle E$

77. $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

78. $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$

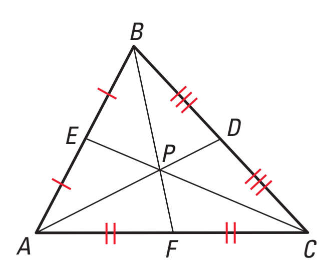
CONCURRENCY PROPERTY FOR MEDIANS **Use the information given in the diagram to fill in the blanks.** (Review 5.3)

79. $AP = 1$, $PD = \underline{?}$

80. $PC = 6.6$, $PE = \underline{?}$

81. $PB = 6$, $FB = \underline{?}$

82. $AD = 39$, $PD = \underline{?}$



83. **INDIRECT PROOF** Write an indirect proof to show that there is no quadrilateral with four acute angles. (Review 6.1 for 6.5)